Chapter 3. Matrices

Matrix and Operations of Matrices

1 Mark Questions

1. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find $(x - y)$. **Delhi 2014**

Given, $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

On comparing the corresponding elements, we get

$$8 + y = 0 \text{ and } 2x + 1 = 5$$

$$\Rightarrow \quad y = -8 \text{ and } x = \frac{5-1}{2} = 2$$

$$\therefore \quad x - y = 2 - (-8) = 10$$ **(1)**

2. Solve the following matrix equation for x.

$$[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$$ **Delhi 2014**

We have, $[x\ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}=0$

By using matrix multiplication, we get

$$[x-2 \quad 0]=[0 \quad 0]$$

On comparing the corresponding elements from both sides, we get

$$x-2=0 \Rightarrow x=2 \qquad \textbf{(1)}$$

3. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix. **All India 2014**

We have, $A^2 = A$

Now,

$7A - (I + A)^3 = 7A - [I^3 + A^3 + 3IA\ (I + A)]$

$[\because (x + y)^3 = x^3 + y^3 + 3xy\ (x + y)]$

$= 7A - [I + A^2 \cdot A + 3A\ (I + A)] \qquad [\because I^3 = I]$

$= 7A - [I + A \cdot A + 3AI + 3A^2] \ [\because A^2 = A, \text{ given}]$

$= 7A - [I + A + 3A + 3A] \qquad [\because AI = A]$

$= 7A - [I + 7A] = - I$ **(1)**

4. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix}=\begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then find the value of $x + y$. **All India 2014**

We have, $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

On comparing the corresponding elements, we get

$$x - y = -1 \quad \text{...(i)}$$

and $$2x - y = 0 \quad \text{...(ii)}$$

On solving the above equations, we get

$$x = 1$$

and $$y = 2$$

Now, $x + y = 1 + 2 = 3$ **(1)**

5. If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$, write the value of $a - 2b$. **Foreign 2014**

Given, $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$

We know that two matrices are equal, if its corresponding elements are equal.

$\therefore$ $$a + 4 = 2a + 2 \quad \text{... (i)}$$

$$3b = b + 2 \quad \text{...(ii)}$$

and $$-6 = a - 8b \quad \text{...(iii)}$$

On solving Eqs. (i), (ii) and (iii), we get

$$a = 2 \quad \text{and} \quad b = 1$$

Now, $a - 2b = 2 - 2(1) = 2 - 2 = 0$ **(1)**

6. If $\begin{bmatrix} x \cdot y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x+y+z)$. **Delhi 2014C**

Given, $\begin{bmatrix} x \cdot y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

We know that, if two matrices are equal, then their corresponding elements are equal.

$\therefore \quad x \cdot y = 8 \Rightarrow y = \frac{8}{x} \quad \ldots(i)$

$z + 6 = 0 \Rightarrow z = -6 \quad \ldots(ii)$

and $\quad x + y = 6 \quad \ldots(iii)$

(1/2)

Now, put the value of y from Eq. (i), in Eq. (iii), we get

$$x + \frac{8}{x} = 6$$

$$\Rightarrow \quad x^2 + 8 = 6x$$

$$\Rightarrow \quad (x-4)(x-2) = 0$$

$$\Rightarrow \quad x = 4, 2$$

On putting the values of x in Eq. (iii), we get

$$y = 2, 4$$

Now, $\quad (x + y + z) = (2 + 4 - 6) = 0 \quad$ **(1/2)**

7. The elements a_{ij} of a 3×3 matrix are given by $a_{ij} = \frac{1}{2}|-3i + j|$. Write the value of element a_{32}. **All India 2014C**

Given, for a 3×3 matrix.

$$a_{ij} = \frac{1}{2}|-3i + j|$$

Here, element a_{32} denotes the element of third row corresponding to second column.

So, to find a_{32}, put $i = 3$ and $j = 2$, we get

$$a_{32} = \frac{1}{2}|-3 \times 3 + 2|$$

$$= \frac{1}{2}|-9 + 2|$$

$$= \frac{7}{2} \quad \textbf{(1)}$$

8. If $[2x \quad 4]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, find the positive value of x. **All India 2014C**

We have, $[2x \ 4]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$

$\Rightarrow \quad (2x^2 - 32) = 0$

$\Rightarrow \quad 2x^2 = 32$

$\Rightarrow \quad x^2 = 16$

$\Rightarrow \quad x = \pm 4$

$\therefore$ Positive value of $x = 4$. **(1)**

9. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then find the value of $(x + y)$.

Delhi 2013C; All India 2012

Firstly, multiply each element of the first matrix by 2, then use property of matrix addition and equality of matrices, to calculate the values of x and y.

Given, $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ **(1/2)**

On comparing corresponding elements, we get $2 + y = 5$ and $2x + 2 = 8$

$\Rightarrow \quad y = 3$ and $2x = 6$

$\Rightarrow \quad y = 3$ and $x = 3$

$\therefore \quad x + y = 3 + 3 = 6$ **(1/2)**

10. Find the value of a, if

$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$. **Delhi 2013**

Use the definition of equality of matrices.

We know that two matrices are equal, if their corresponding elements are equal. **(1/2)**

$\therefore \quad a - b = -1$ ...(i)

and $\quad 2a - b = 0$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$a = 1$ **(1/2)**

11. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A. **Delhi 2013**

Given matrix equation can be rewritten as

$$A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix} \quad \textbf{(1/2)}$$

$$\Rightarrow \quad A = \begin{bmatrix} 9-1 & -1-2 & 4+1 \\ -2-0 & 1-4 & 3-9 \end{bmatrix}$$

[two matrices can be subtracted only when their orders are same]

$$= \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix} \quad \textbf{(1/2)}$$

12. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k. **All India 2013**

Given, $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ ...(i)

and $A^2 = kA$

Now, $A^2 = A \cdot A$ ...(ii)

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix}$$

[multiplying row by column]

$$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \textbf{(1/2)}$$

$\Rightarrow \quad A^2 = 2A$ [from Eq. (i)]

On comparing with Eq. (ii) we get

$k = 2$ **(1/2)**

13. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then write the value of p. **All India 2013**

Given, $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ ...(i)

and $A^2 = pA$ ...(ii)

Now, $A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix}$$

[multiplying row by column]

$$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$ **(1/2)**

$$= 4 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$\Rightarrow$ $A^2 = 4A$ [from Eq.(i)]

On comparing with Eq. (ii), we get

$$p = 4$$ **(1/2)**

14. If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ. **All India 2013**

Given, matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ ...(i)

Also, $A^2 = \lambda A$ ...(ii)

Now, $A^2 = A \cdot A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 9+9 & -9-9 \\ -9-9 & 9+9 \end{bmatrix}$$

[multiplying row by column]

$$= \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \begin{bmatrix} 6\cdot 3 & -6\cdot 3 \\ -6\cdot 3 & 6\cdot 3 \end{bmatrix}$$

$$= 6\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$ **(1/2)**

$\Rightarrow \quad \lambda A = 6A$ [from Eqs. (i) and (ii)]

$\therefore \quad \lambda = 6$ **(1/2)**

15. Simplify

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}.$$

Delhi 2012; HOTS

> Firstly, we multiply each element of the first matrix by $\cos\theta$ and second matrix by $\sin\theta$ and then using the matrix addition.

We have,

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= I =$ unit matrix **(1)**

16. Find the value of $y - x$ from following equation

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}.$$

All India 2012

We have,

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} = \begin{bmatrix} 7-3 & 6+4 \\ 15-1 & 14-2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 \\ 14 & 12 \end{bmatrix}$$ **(1/2)**

On equating the corresponding elements, we get

$$2x = 4 \text{ and } 2y - 6 = 12$$

$$\Rightarrow \quad x = 2 \text{ and } 2y = 18$$

$$\Rightarrow \quad x = 2 \text{ and } y = 9$$

$$\therefore \quad y - x = 9 - 2 = 7$$ **(1/2)**

17. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, then write the value of x.

Foreign 2012

We have, $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

On comparing corresponding elements, we get

$$2x-y=10,\ 3x+y=5$$

On adding both equations, we get

$$5x=15 \Rightarrow x=3 \qquad \textbf{(1)}$$

18. If $3A-B=\begin{bmatrix}5&0\\1&1\end{bmatrix}$ and $B=\begin{bmatrix}4&3\\2&5\end{bmatrix}$, then find the matrix A. **Delhi 2012C**

Given $B=\begin{bmatrix}4&3\\2&5\end{bmatrix}$ and $3A-B=\begin{bmatrix}5&0\\1&1\end{bmatrix}$

$\Rightarrow \quad 3A=\begin{bmatrix}5&0\\1&1\end{bmatrix}+B=\begin{bmatrix}5&0\\1&1\end{bmatrix}+\begin{bmatrix}4&3\\2&5\end{bmatrix}$

$$\left[\text{put } B=\begin{bmatrix}4&3\\2&5\end{bmatrix}\right]$$

$$=\begin{bmatrix}5+4&3\\1+2&1+5\end{bmatrix}=\begin{bmatrix}9&3\\3&6\end{bmatrix}=3\begin{bmatrix}3&1\\1&2\end{bmatrix}$$

On comparing both sides, we get

$$A=\begin{bmatrix}3&1\\1&2\end{bmatrix} \qquad \textbf{(1)}$$

19. Write the value of $x-y+z$ from following equation

$$\begin{bmatrix}x+y+z\\x+z\\y+z\end{bmatrix}=\begin{bmatrix}9\\5\\7\end{bmatrix}.$$ **Foreign 2011**

Use the definition of equality of matrices i.e. if two matrices are equal, then their corresponding elements are equal.

Given matrix equation is

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

On equating the corresponding elements, we get

$$x+y+z=9 \quad \ldots(i)$$
$$x+z=5 \quad \ldots(ii)$$

and $$y+z=7 \quad \ldots(iii)$$

On putting the value of $x+z$ from Eq. (ii) in Eq. (i), we get

$$y+5=9 \Rightarrow y=4$$

On putting $y=4$ in Eq. (iii), we get $z=3$

Again, putting $z=3$ in Eq. (ii), we get $x=2$

Now, $x-y+z=2-4+3=1$ **(1)**

20. Write the order of product matrix

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4].$$ **Foreign 2011; HOTS**

Use the fact that if a matrix A has order $m \times n$ and other matrix B has order $n \times z$, then the matrix AB has order $m \times z$, that means if number of columns of matrix A is same as number of rows of matrix B, then matrix multiplication AB is possible.

Let $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [2 \quad 3 \quad 4]$

Here, order of matrix $A = 3 \times 1$

and order of matri x $B = 1 \times 3$

$\therefore$ Order of product matrix $AB = 3 \times 3$ **(1)**

21. If a matrix has 5 elements, then write all possible orders it can have. **All India 2011**

Use the result that a matrix has order $m \times n$, then total number of elements in that matrix is mn.

Given a matrix has 5 elements. So, possible order of this matrix are $5 \times 1, 1 \times 5$. **(1)**

22. For a 2×2 matrix, $A = [a_{ij}]$ whose elements are given by $a_{ij} = i/j$, write the value of a_{12}.

Delhi 2011

Given, for a 2×2 matrix,

$$A = [a_{ij}],\ a_{ij} = \frac{i}{j}$$

To find a_{12}, put $i = 1$ and $j = 2$, we get

$$a_{12} = \frac{1}{2} \qquad \textbf{(1)}$$

23. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y. Delhi 2011C

Given, $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$

On comparing corresponding elements, we get

$x = 3$ and $x - y = 1 \Rightarrow y = x - 1 = 3 - 1 = 2$ **(1)**

24. From the following matrix equation, find the value of x.

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$ Foreign 2010

Do same as Que 10. **[Ans.** 1]

25. Find x from the matrix equation

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$ Foreign 2010; HOTS

Firstly, we calculate the multiplication of matrices in LHS and then equate the corresponding elements of both sides.

Given matrix equation is $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x+6 \\ 4x+10 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

[multiplying row by column]

On equating the corresponding elements, we get

$$x + 6 = 5$$

$$\Rightarrow \quad x = -1 \qquad \textbf{(1)}$$

26. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$, then find the value of x.

Foreign 2010; HOTS

Do same as Que 25. [**Ans.** 5]

27. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix? Delhi 2010; HOTS

Firstly, we put the given matrix A equal to an identity matrix and then equate the corresponding elements to get the value of α.

Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

For A to be an identity matrix, we must have

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \left[\because I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$$

On equating element a_{11} from both sides, we get

$$\cos\alpha = 1$$

$$\Rightarrow \quad \cos\alpha = \cos 0^\circ \qquad [\because \cos 0^\circ = 1]$$

$$\therefore \quad \alpha = 0^\circ$$

So, for $\alpha = 0^\circ$, A is an identity matrix.

$[\because \sin 0^\circ = 0]$ **(1)**

28. If $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 5\end{bmatrix}=\begin{bmatrix}7 & 11\\k & 23\end{bmatrix}$, then write the value of k. Delhi 2010

Given,

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 5\end{bmatrix}=\begin{bmatrix}7 & 11\\k & 23\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}3+4 & 1+10\\9+8 & 3+20\end{bmatrix}=\begin{bmatrix}7 & 11\\k & 23\end{bmatrix}$$

[multiplying row by column]

$$\Rightarrow \quad \begin{bmatrix}7 & 11\\17 & 23\end{bmatrix}=\begin{bmatrix}7 & 11\\k & 23\end{bmatrix}$$

On equating element a_{21} from both sides, we get

$$17=k$$

$$\Rightarrow \quad k=17 \qquad \textbf{(1)}$$

29. If A is a matrix of order 3×4 and B is a matrix of order 4×3, then find order of matrix (AB). Delhi 2010C

Order of matrix $AB=3\times3$

[if a matrix A has order $x\times y$ and B has order $y\times z$, then matrix AB has order $x\times z$] **(1)**

30. If $\begin{bmatrix}x+y & 1\\2y & 5\end{bmatrix}=\begin{bmatrix}7 & 1\\4 & 5\end{bmatrix}$, then find the value of x. Delhi 2010C

Given matrix equation is $\begin{bmatrix} x+y & 1 \\ 2y & 5 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$

On equating the corresponding elements, we get

$$x + y = 7 \quad \text{...(i)}$$

and

$$2y = 4 \quad \text{...(ii)}$$

From Eq. (ii), we get

$$y = \frac{4}{2} = 2$$

On putting the value of y in Eq. (i), we get

$$x + 2 = 7$$

$$\Rightarrow \quad x = 5$$ **(1)**

31. If $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$, then find the value of x. All India 2010C

Do same as Que 30. [**Ans.** $x = 3$]

32. If $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, then find the value of y. All India 2010C

Do same as Que 30. [**Ans.** $y = 2$]

33. If $\begin{bmatrix} 2x & 1 \\ 5 & x+2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$, then find the value of y. All India 2009C

Do same as Que 30. [**Ans.** $y = -1$]

34. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then find the value of y. Foreign 2009

Do same as Que 30. [**Ans.** $y = 3$]

35. Find the value of x, if

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}.$$

All India 2009

Do same as Que 30. [**Ans.** $x = 1$]

NOTE Sometimes on solving an equation, we get more than one values of one variable. This means that such a matrix does not exist.

36. Find the value of y, if $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$.

All India 2009

Do same as Que 30. [**Ans.** $y = 1$]

37. Find the value of x, if $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$.

All India 2009

Do same as Que 30. [**Ans.** $x = 2$]

38. If $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$, then find the value of x.

Delhi 2009C

Do same as Que 30. [**Ans.** $x = 5$]

39. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find $A - B$.

All India 2008C

For finding $A - B$, subtracting the corresponding elements.

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$\therefore \quad A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix} \quad \textbf{(1)}$$

40. If $\begin{bmatrix} x+2y & 3y \\ 4x & 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 2 \end{bmatrix}$, then find x and y.
All India 2008C

Do same as Que 30. **[Ans.** $x = 2, y = 1$]

41. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.
Delhi 2008; HOTS

Do same as Que 9. **[Ans.** $x = 3, y = 3$]

4 Marks Questions

42. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find value of $A^2 - 3A + 2I$.
All India 2010

Given, $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

We have to find the value of $A^2 - 3A + 2I$.

Now, $A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

[multiplying row by column]

$$\Rightarrow \quad A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} \qquad \textbf{(1½)}$$

$$3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} \qquad \textbf{(1/2)}$$

$$\text{and} \quad 2I = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \textbf{(1/2)}$$

$$\therefore \quad A^2 - 3A + 2I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^2 - 3A + 2I$$

$$= \begin{bmatrix} 5-6+2 & -1-0+0 & 2-3+0 \\ 9-6+0 & -2-3+2 & 5-9+0 \\ 0-3+0 & -1+3+0 & -2-0+2 \end{bmatrix}$$

$$\Rightarrow \quad A^2 - 3A + 2I = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix} \qquad \textbf{(1½)}$$

43. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that $A^2 - 4A - 5I = O$. Delhi 2008

Given, $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$ **(1½)**

Now, LHS $= A^2 - 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(1)**

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{RHS}$$ **(1½) Hence proved.**

Transpose of a Matrix and Symmetric Matrix

1 Mark Questions

1. Write 2×2 matrix which is both symmetric and skew-symmetric matrices. **Delhi 2014C**

A null matrix of order 2×2 is both symmetric and skew-symmetric matrices.

For a symmetric matrix,

$$a_{ij} = a_{ji} \quad \ldots(i)$$

and for a skew-symmetric matrix,

$$a_{ij} = -a_{ji} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get $a_{ij} = 0$ **(1)**

2. For what value of x, is the matrix

$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

All India 2013; HOTS

If A is a skew-symmetric matrix, then $A = -A^T$, where A^T is transpose of matrix A.

Given, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$

We know that, if A is a skew-symmetric matrix, then

$$A = -A^T \qquad \text{...(i)}$$

From Eq. (i), we get

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \qquad \textbf{(1/2)}$$

On comparing the corresponding element, we get

$$x = 2 \qquad \textbf{(1/2)}$$

3. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

All India 2012

Given, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Transpose of $B = B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ **(1/2)**

Now, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ **(1/2)**

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $A + A'$.

All India 2010C

Firstly, we find the transpose of matrix A and then add the corresponding elements of both matrices A and A'.

Given, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore \quad A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ **(1)**

5. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, then find $A + A'$, where A' is transpose of A.

All India 2009C

Do same as Que 4. $\left[\textbf{Ans. } \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}\right]$

6. If matrix $A = [1 \quad 2 \quad 3]$, then write AA'.

Delhi 2009; HOTS

Firstly, we write the transpose of matrix A of order 1×3, whose order is 3×1, then multiply if matrix multiplication is possible to get required answer.

Given, matrix is $A = [1 \quad 2 \quad 3]$

$$\therefore \qquad A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{Now,} \qquad AA' = [1 \quad 2 \quad 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= [(1 \times 1) + (2 \times 2) + (3 \times 3)]$$

$$= [1 + 4 + 9] = [14] \qquad \textbf{(1)}$$

4 Mark Questions

7. For the following matrices A and B, verify that

$$[AB]' = B'A'; \quad A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1 \quad 2 \quad 1].$$

All India 2010

Given, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1 \quad 2 \quad 1]$

To verify $(AB)' = B'A'$

$$\text{Here,} \qquad AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3 \times 1} [-1 \quad 2 \quad 1]_{1 \times 3}$$

$$\Rightarrow \qquad AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

[multiplying row by column]

$$\therefore \qquad \text{LHS} = (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \qquad \ldots\text{(i)}$$

[interchanging rows and columns] **(1½)**

$$\text{Now,} \qquad B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \text{ and } A' = [1 \quad -4 \quad 3] \qquad \textbf{(1)}$$

$$\therefore \qquad \text{RHS} = B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \quad -4 \quad 3]$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \qquad \ldots\text{(ii)}$$

[multiplying row by column]**(1)**

From Eqs. (i) and (ii), we get

$$(AB)' = B'A'$$

$$\therefore \qquad \text{LHS} = \text{RHS} \qquad \textbf{(1/2)}$$

8. Express the following matrix as a sum of a symmetric and a skew-symmetric matrices and verify your result.

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$ **All India 2010; HOTS**

Write the given matrix A as $A = P + Q$, where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$. Also, verify that P is a symmetric matrix and Q is a skew-symmetric matrix.

Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Let us introduce two matrices P and Q, such that

$$P = \frac{1}{2}(A + A') \text{ and } Q = \frac{1}{2}(A - A')$$

We will show that $A = (P + Q)$

Now, $P = \frac{1}{2}(A + A')$

$$\Rightarrow P = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} \quad \textbf{(1)}$$

and $\quad P' = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = P$

$\therefore \quad P' = P$

So, P is a symmetric matrix. **(1)**

Now, $Q = \frac{1}{2}(A - A')$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

and $Q' = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ \ldots \end{bmatrix}$

$$\text{and } Q' = \frac{1}{2}\begin{bmatrix} -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= -Q$$

$$\Rightarrow \quad Q' = -Q$$

$\therefore$ Q is a skew-symmetric matrix. **(1)**

Now,

$$P + Q = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & -4 & -8 \\ 6 & -4 & -10 \\ -2 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Thus, $P + Q = A$ **(1)**

Inverse of a Matrix by Elementary Operations

Previous Year Examination Question

4 Marks Questions

1. Use elementary column operations $C_2 \to C_2 - 2C_1$ in the matrix equation

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}.$$

Foreign 2014

> We write the matrix A as $A = AI$ for applying elementary column operations. So, apply column operation on the matrix of LHS and on the second matrix of RHS.

Given matrix equation is

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

On applying $C_2 \to C_2 - C_1$, we get

$$\begin{bmatrix} 4 & 2-8 \\ 3 & 3-6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0-4 \\ 1 & 1-2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

which is the required answer.

2. Using elementary row transformation (ERT), find inverse of matrix $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Foreign 2010; HOTS

Firstly, put $A = IA$. Then, by applying elementary row transformation on A of LHS and I of RHS, convert this matrix in the form $I = BA$, where B gives the inverse of A.

Given matrix is $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_2 \to R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Now, applying $R_2 \to (-1)R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Hence, $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$ $[\because A^{-1}A = I]$ **(1/2)**

3. Find A^{-1}, by using elementary row transformation for matrix $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$.

Foreign 2010

Do same as Que 2. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\right]$

4. Using elementary row transformation, find inverse of matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$. **Delhi 2010**

Do same as Que 2. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\right]$

6 Marks Questions

5. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ **Delhi 2012**

Given matrix is $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Let $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 3R_1$, we get

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to (-1)\, R_1$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_2 \to (-1) R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2 R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_3 \to (-1) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

which is of the form $I = BA$.

Hence, $$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \qquad \textbf{(1)}$$

6. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Foreign 2011

. Given matrix is $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Let $$A = IA$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to 3R_1$, we get

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_2 \to R_2 - 5R_1$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \textbf{(1)}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Applying $R_3 \to \frac{1}{3} R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \qquad \textbf{(1/2)}$$

Hence, $$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \qquad \textbf{(1)}$$

7. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Delhi 2010

Given matrix is $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$. **(1)**

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \quad \textbf{(1)}$$

Applying $R_2 \to \dfrac{R_2}{9}$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \quad \mathbf{(1/2)}$$

Applying $R_3 \to R_3 + 5R_2$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \quad \mathbf{(1/2)}$$

Applying $R_3 \to 9R_3$, we get

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/9 & 0 \\ -3 & 5 & 9 \end{bmatrix} A \quad \mathbf{(1/2)}$$

Applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0 \\ -3 & 5 & 9 \end{bmatrix} A \quad \mathbf{(1/2)}$$

Applying $R_1 \to R_1 - \dfrac{1}{3}R_3$ and $R_2 \to R_2 + \dfrac{7}{9}R_3$,

we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

Hence, $$A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} \quad \mathbf{(1)}$$

8. $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

All India 2010

Do same as Que 7. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\right]$

9. $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

All India 2009

Do same as Que 7. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}\right]$

10. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

All India 2008

Do same as Que 7. $\left[\textbf{Ans. } A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}\right]$